

Perfect Numbers?

17

Facts and Reminders

Proper Factors

The *proper factors* of a number are all of the factors of a number except the number itself.

Sample A

The factors of 14 are 1, 2, 7, and 14. The proper factors are 1, 2, and 7. (*Note:* The number itself, 14, is not included in the proper factors.)

Sample B

The factors of 9 are 1, 3, and 9. (*Note:* The 3 is not repeated.) The proper factors are 1 and 3.

Numbers may be designated perfect, defective, or abundant based on the sum of their proper factors.

Perfect Numbers

A *perfect number* is equal to the sum of all of its factors, except the number itself. Read the example below.

Sample

The proper factors of 6 are: 1, 2, and 3. Add $1 + 2 + 3 = 6$. Therefore, 6 is a perfect number.

There are no known odd perfect numbers. Perfect numbers are very rare.

Abundant Numbers

Abundant numbers are those numbers where the sum of the proper factors is greater than the number itself.

Sample

The proper factors of 12 are 1, 2, 3, 4, and 6. Add $1 + 2 + 3 + 4 + 6$. The sum, 16, is greater than 12 and 12 is, therefore, abundant. The number 12 is the first abundant number.

There are only 21 abundant numbers between 12 and 100.

Defective (Deficient) Numbers

Defective numbers are those numbers in which the sum of the proper factors is less than the number itself. Defective numbers are sometimes called *deficient numbers*.

Sample

The proper factors of 22 are 1, 2, and 11. Add $1 + 2 + 11$. The sum, 14, is less than 22 and 22 is, therefore, defective.

Most numbers are defective because they have very few factors. All prime numbers are defective.

Perfect Numbers?

Proper Factors

What are the proper factors of 15? factors: 1, 3, 5, 15 proper factors: 1, 3, 5

Directions: Study the sample above and the Facts and Reminders page for this unit. List the proper factors for each number listed below.

1. 21

factors: _____
proper factors: _____

2. 25

factors: _____
proper factors: _____

3. 18

factors: _____
proper factors: _____

4. 20

factors: _____
proper factors: _____

5. 28

factors: _____
proper factors: _____

6. 16

factors: _____
proper factors: _____

7. 36

factors: _____
proper factors: _____

8. 44

factors: _____
proper factors: _____

Directions: Use a calculator to compute the sum of the proper factors for each number listed below.

9. 72

factors: _____
proper factors: _____
sum of proper factors: _____

10. 81

factors: _____
proper factors: _____
sum of proper factors: _____

11. 100

factors: _____
proper factors: _____
sum of proper factors: _____

12. 144

factors: _____
proper factors: _____
sum of proper factors: _____

13. 200

factors: _____
proper factors: _____
sum of proper factors: _____

14. 98

factors: _____
proper factors: _____
sum of proper factors: _____

Perfect Numbers?

Working with Abundant and Defective Numbers

Is 24 an abundant or defective number?

The proper factors of 24 are 1, 2, 3, 4, 6, 8, and 12.

The sum of the proper factors is 36. So 24 is an abundant number.

Directions: Study the sample above and the Facts and Reminders page for this unit. List the proper factors for each number listed below. Use a calculator to compute the sum of the proper factors for each number listed below. Label each number as *abundant* or *defective*.

1. 20

proper factors: _____

sum of proper factors: _____

name: _____

2. 16

proper factors: _____

sum of proper factors: _____

name: _____

3. 38

proper factors: _____

sum of proper factors: _____

name: _____

4. 50

proper factors: _____

sum of proper factors: _____

name: _____

5. 64

proper factors: _____

sum of proper factors: _____

name: _____

6. 18

proper factors: _____

sum of proper factors: _____

name: _____

7. 100

proper factors: _____

sum of proper factors: _____

name: _____

8. 36

proper factors: _____

sum of proper factors: _____

name: _____

9. 48

proper factors: _____

sum of proper factors: _____

name: _____

10. 144

proper factors: _____

sum of proper factors: _____

name: _____

11. 125

proper factors: _____

sum of proper factors: _____

name: _____

12. 150

proper factors: _____

sum of proper factors: _____

name: _____

13. 60

proper factors: _____

sum of proper factors: _____

name: _____

14. 90

proper factors: _____

sum of proper factors: _____

name: _____

Perfect Numbers?

Determining Perfect, Abundant, and Defective Numbers

Is 28 an abundant, defective, or perfect number?

The proper factors of 28 are 1, 2, 4, 7, and 14.

The sum of the proper factors is 28. So 28 is a perfect number.

Directions: Study the Facts and Reminders page for this unit. List the proper factors for each number listed below. Use a calculator to compute the sum of the proper factors for each number listed below. Label each number as *abundant*, *defective*, or *perfect*.

1. 116

proper factors: _____

sum of proper factors: _____

name: _____

2. 300

proper factors: _____

sum of proper factors: _____

name: _____

3. 95

proper factors: _____

sum of proper factors: _____

name: _____

4. 380

proper factors: _____

sum of proper factors: _____

name: _____

5. 120

proper factors: _____

sum of proper factors: _____

name: _____

6. 288

proper factors: _____

sum of proper factors: _____

name: _____

7. 496

proper factors: _____

sum of proper factors: _____

name: _____

8. 888

proper factors: _____

sum of proper factors: _____

name: _____

9. 960

proper factors: _____

sum of proper factors: _____

name: _____

10. 100

proper factors: _____

sum of proper factors: _____

name: _____

11. 498

proper factors: _____

sum of proper factors: _____

name: _____

12. 900

proper factors: _____

sum of proper factors: _____

name: _____

13. 1,000

proper factors: _____

sum of proper factors: _____

name: _____

14. 999

proper factors: _____

sum of proper factors: _____

name: _____

Answer Key

12. $6 \frac{1}{10}$
13. $13 \frac{2}{3}$
14. $12 \frac{11}{20}$

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1. $1 \frac{5}{8}$
2. $1 \frac{1}{4}$
3. $6 \frac{1}{6}$
4. $3 \frac{11}{20}$
5. $6 \frac{1}{8}$
6. $3 \frac{1}{3}$
7. $6 \frac{1}{4}$
8. $4 \frac{1}{9}$
9. $2 \frac{5}{6}$
10. $2 \frac{3}{4}$
11. $1 \frac{7}{8}$
12. $2 \frac{5}{6}$
13. $3 \frac{19}{20}$
14. $4 \frac{17}{20}$

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1. 6
2. $7 \frac{1}{3}$
3. 3
4. 6
5. 4
6. 6
7. $\frac{7}{8}$
8. $\frac{1}{2}$
9. $\frac{1}{4}$
10. $\frac{1}{2}$
11. $\frac{4}{5}$
12. $1 \frac{1}{2}$

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1. 1, 3, 7, 21
1, 3, 7
2. 1, 5, 25
1, 5
3. 1, 2, 3, 6, 9, 18
1, 2, 3, 6, 9
4. 1, 2, 4, 5, 10, 20
1, 2, 4, 5, 10
5. 1, 2, 4, 7, 14, 28
1, 2, 4, 7, 14
6. 1, 2, 4, 8, 16
1, 2, 4, 8
7. 1, 2, 3, 4, 6, 9, 12, 18, 36
1, 2, 3, 4, 6, 9, 12, 18
8. 1, 2, 4, 11, 22, 44
1, 2, 4, 11, 22
9. 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
1, 2, 3, 4, 6, 8, 9, 12,

- 18, 24, 36
123
10. 1, 3, 9, 27, 81
1, 3, 9, 27
40
11. 1, 2, 4, 5, 10, 20, 25, 50, 100
1, 2, 4, 5, 10, 20, 25, 50
117
12. 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72
259
13. 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100
265
14. 1, 2, 7, 14, 49, 98
1, 2, 7, 14, 49
73

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1. 1, 2, 4, 5, 10
22
abundant
2. 1, 2, 4, 8
15
defective
3. 1, 2, 19
22
defective
4. 1, 2, 5, 10, 25
43
defective
5. 1, 2, 4, 8, 16, 32
63
defective
6. 1, 2, 3, 6, 9
21
abundant
7. 1, 2, 4, 5, 10, 20, 25, 50
117
abundant
8. 1, 2, 3, 4, 6, 9, 12, 18
55
abundant
9. 1, 2, 3, 4, 6, 8, 12, 16, 24

- 76
abundant
10. 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72
259
abundant
11. 1, 5, 25
31
defective
12. 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75
222
abundant
13. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30
108
abundant
14. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45
144
abundant

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1. 1, 2, 4, 29, 58
94
defective
2. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150
568
abundant
3. 1, 5, 19
25
defective
4. 1, 2, 4, 5, 10, 19, 20, 38, 76, 95, 190
460
abundant
5. 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60
240
abundant
6. 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, 36, 48, 72, 96, 144
531
abundant
7. 1, 2, 4, 8, 16, 31, 62, 124, 248
496
perfect
8. 1, 2, 4, 6, 8, 12, 24,

- 37, 74, 111, 148, 222, 444
1093
abundant
9. 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64, 80, 96, 120, 160, 192, 240, 320, 480
2088
abundant
10. 1, 2, 4, 5, 10, 20, 25, 50, 100
217
abundant
11. 1, 2, 3, 6, 83, 166, 249
510
abundant
12. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 30, 36, 45, 50, 60, 75, 90, 100, 150, 180, 225, 300, 450
1921
abundant
13. 1, 2, 4, 5, 10, 20, 25, 40, 50, 100, 200, 250, 500
1207
abundant
14. 1, 3, 9, 27, 37, 111, 333
521
defective

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1. terminating, 0.375
2. non-terminating, 0.3333
3. terminating, 0.875
4. non-terminating, 0.4444
5. terminating, 0.800
6. non-terminating, 0.571428571
7. terminating, 0.750
8. non-terminating, 0.3333
9. non-terminating, 0.272727