Facts and Reminders

Ordering Two Objects

Place one red crayon and one blue crayon on your desk. How many different ways can you arrange them?

| red | blue | | blue | red |

There are two different ways. You can put the red crayon first and the blue second, or you can put the blue first and the red second.

Ordering Three Objects

Place a third crayon, a green, on your desk. How many different ways can you arrange them?

red	blue	green
red	green	blue
green	red	blue
green	blue	red
blue	green	red
blue	red	green
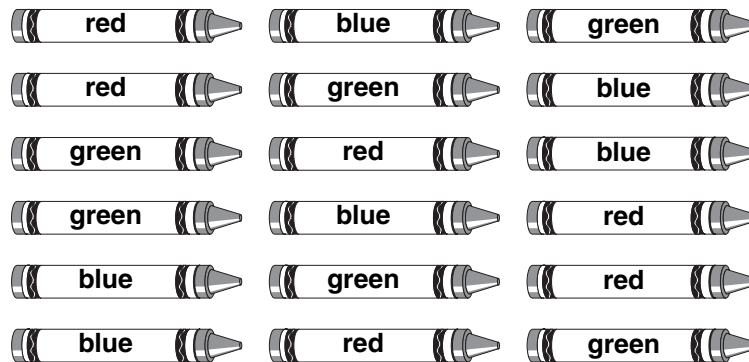
There are six different ways as shown in the diagram. Use your crayons to see if any arrangement has been missed. Notice that red is first in two arrangements, second in two arrangements, and third in two arrangements. This is true for each of the other colors as well.

Factorials

These arrangements can be expressed by a math concept called a *factorial*. The two-color arrangement is expressed this way: $2! = 2 \times 1 = 2$

The three-color factorial is expressed this way: $3! = 3 \times 2 \times 1 = 6$

The exclamation point (!) is the sign for a factorial. Factorials can be multiplied:

$$2! \times 3! = 2 \times 1 \times 3 \times 2 \times 1 = 12$$

Factorials can be divided:

$$\frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$$

Fancy Factorials

Working with Simple Factorials

Directions: Study the Facts and Reminders page for this unit. Place four crayons—one red, one blue, one green, and one yellow—on your desk. Find out how many different ways you can arrange them. Complete the chart below.

red	blue	green	yellow
red	blue	yellow	green
red	green	blue	yellow
red	green	yellow	blue
red	yellow	green	blue
red	yellow	blue	green
blue	red	green	yellow
blue	red	yellow	green
blue	green	red	yellow
blue	green	yellow	red
blue	yellow	green	red
blue	yellow	red	green
green	blue	red	yellow
green	blue	yellow	_____
green	red	_____	_____
green	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
yellow	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
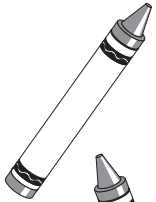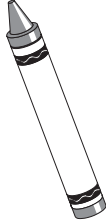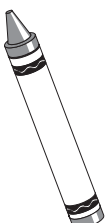
Directions: Use a factorial to help you answer these questions.

1. You received three trophies: one for soccer, one for baseball, and one for track. How many different ways could you arrange these three trophies on your dresser? _____

2. You have four coins: one dime, one nickel, one penny, and one quarter. How many different ways can you arrange them in order on your desk? _____

3. You have a pencil and a pen. How many different ways can you arrange them in order in your shirt pocket? _____

Fancy Factorials

Working with Larger Factorials

Directions: Study the Facts and Reminders page for this unit. Place five crayons—one red, one blue, one green, one yellow, and one orange—on your desk. Find out how many different ways you can arrange them. Fill in the blanks on this chart. (*Note:* R = red, B = blue, G = green, Y = yellow, and O = orange)

R	B	G	Y	O
R	B	G	O	Y
R	B	Y	O	G
R	B	Y	G	O
R	B	O	Y	G
R	B	O	G	Y
R	G	B	Y	O
R	G	B	O	Y
R	G	Y	O	B
R	G	Y	B	O
R	G	O	Y	B
R	G	O	B	Y
R	Y	G	B	O
R	Y	G	O	B
R				
R				
R				
R				
R	O			
R	O			
R				
R				
R				
R				

1. This chart shows only the arrangements with red as the first color. How many different arrangements would you estimate for all five colors?

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = $$

2. Write a factorial to indicate how many ways six crayons could be arranged. Then compute the numerical value of 6!.

3. Write a factorial to indicate how many ways seven crayons could be arranged.

Fancy Factorials

Computing Larger Factorials

Directions: Write out each factorial as a multiplication problem. Use a calculator to help you compute the numerical values for these factorials. The first one is started for you.

1. 6! = 6 x 5 x 4 x 3 x 2 x 1   numerical value: _____

2. 7! = _____   numerical value: _____

3. 8! = _____   numerical value: _____

4. 9! = _____   numerical value: _____

5. 10! = _____   numerical value: _____

Directions: Study the Facts and Reminders page for this unit. Write out each factorial as a multiplication problem. Use a calculator to help you compute the numerical values for these factorials. The first one is started for you.

6. 2! x 3! = 2 x 1 x 3 x 2 x 1 =

     numerical value: _____

7. 3! x 4! = _____

     numerical value: _____

8. 2! x 5! = _____

     numerical value: _____

9. 6! x 3! = _____

     numerical value: _____

Directions: Use the Facts and Reminders sheet to help you divide these factorials. Cancel numbers which are the same above and below the fraction bar. The first one is started for you.

10. $\frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1}$ =

11. $\frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$ =

12. $\frac{9!}{7!} =$ _____ =

13. $\frac{10!}{9!} =$ _____ =

14. $\frac{10!}{4!} =$ _____ =

15. $\frac{12!}{8!} =$ _____ =

10. 3/4
11. 4/5
12. 3/4
13. 3/4
14. 2/3
15. 5/6
16. 2/5
17. 1/3
18. 5/6
19. 3/4
20. 2/3
21. 5/13
22. 11/17
23. 4/5
24. 2/3
25. 3/5
26. 7/19
27. 17/23
28. 8/11
29. 31/51
30. 4/7
31. 7/9
32. 18/19

Page 24
1. 3/5
2. 3/4
3. 5/8
4. 7/12
5. 5/8
6. 3/9 = 1/3
7. 7/7 = 1
8. 2/8 = 1/4
9. 6/10 = 3/5
10. 20
11. 9
12. 12
13. 6
14. 24
15. 4
16. 18
17. 22
18. 24
19. 6
20. 35
21. 18
22. 6
23. 24
24. 30
25. 18

Page 25
1. 13/20
2. 3/4
3. 7/8
4. 1/6
5. 4/9
6. 11/12
7. 7/8
8. 9/10
9. 8/9
10. 11/15
11. 3/20
12. 7/24
13. 26/21 = 1 5/21
14. 35/24 = 1 11/24

15. 22/18 = 1 2/9
16. 5/24
17. 13/36
18. 14/72 = 7/36
19. 59/42 = 1 17/42
20. 29/24 = 1 5/24
21. 3/20
22. 23/18 = 1 5/18
23. 55/60 = 11/12
24. 25/30 = 5/6

Page 26
1. 5/6 pie
2. 41/24 = 1 17/24 pizza
3. 5/9 cake
4. 1/6 candy bar
5. 37/24 = 1 13/24 pizza
6. 1/18 cake
7. 7/36 pizza

Page 28
1. 1
2. 1
3. 1
4. 1
5. 1
6. 1
7. 1
8. 1
9. 1
10. 1/8
11. 1/2
12. 1/9
13. 1/3
14. 3/5
15. 1/4
16. 5/9

Page 29
1. 1/12
2. 5/12
3. 2/9
4. 1/9
5. 1/6
6. 11/24
7. 1/9
8. 3/7
9. 1/3
10. 1/6
11. 9/16
12. 3/5
13. 2/5
14. 4/15
15. 4/15
16. 1/6
17. 5/42
18. 3 3/10
19. 3/10
20. 3/64
21. 1/2

Page 30
1. 2
2. 2
3. 2
4. 4
5. 3
6. 4
7. 1 3/4
8. 5/6
9. 7/6 = 1 1/6
10. 9/5 = 1 4/5
11. 7/2 = 3 1/2
12. 1

13. 3
14. 8
15. 1/3

Page 32
(Answers on the chart may be in different order.)
green, blue, yellow, red
green, red, yellow, blue
green, red, blue, yellow
green, yellow, blue, red
green, yellow, red, blue
yellow, blue, red, green
yellow, blue, green, red
yellow, green, blue, red
yellow, green, red, blue
yellow, red, green, blue
yellow, red, blue, green
1. 3! = 3 x 2 x 1 = 6
2. 4! = 4 x 3 x 2 x 1 = 24
3. 2! = 2 x 1 = 2

Page 33
(Answers on the chart may be in different order.)
RYBOG
RYBGO
RYOGB
RYOBG
ROYGB
ROYBG
ROGYB
ROGBY
ROBYG
ROBGY
1. 120 possible arrangements
2. 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720
3. 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5,040

Page 34
1. 720
2. 7 x 6 x 5 x 4 x 3 x 2 x 1
   5,040
3. 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
   40,320
4. 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
   362,880
5. 10 x 9 x 8 x 7 x 6 x 5

x 4 x 3 x 2 x 1
3,628,800
6. 12
7. 3 x 2 x 1 x 4 x 3 x 2 x 1
   144
8. 2 x 1 x 5 x 4 x 3 x 2 x 1
   240
9. 6 x 5 x 4 x 3 x 2 x 1 x 3 x 2 x 1
   4,320
10. 12
11. 20
12. 72
13. 10
14. 151,200
15. 11,880

Page 36
1. 6.908 cm
2. 5.024 cm
3. 8.792 cm
4. 3.14 cm
5. 10.048 cm
6. 9.42 cm
7. 14.444 cm
8. 12.56 cm

Page 37
1. 3.14 sq. cm
2. 1.5386 sq. cm
3. 2.5434 sq. cm
4. 7.065 sq. cm
5. 9.0746 sq. cm
6. 5.3066 sq. cm
7. 12.56 sq. cm
8. 10.1736 sq. cm

Page 38
1. 62.8 m, 314 m²
2. 43.96 m, 153.86 m²
3. 40.82 m, 132.665 m²
4. 157 cm, 1962.5 cm²
5. 128.74 ft., 1319.585 ft.²
6. 15.7 cm, 19.625 cm²
7. 75.36 in., 452.16 in.²
8. 62.8 cm, 314 cm²
9. 37.68 ft., 113.04 ft.²
10. 157 mm, 1962.5 mm²