Secret Sequences 16

Facts and Reminders

Simple Sequences

A *sequence* is a set of numbers which follows a mathematical rule and a specific order. This is a simple sequence:

(1, 2, 3, 4, 5, 6, 7, 8, . . .)

Odd numbers create a sequence following a specific arrangement:

(1, 3, 5, 7, 9, 11, 13, . . .)

Sequences can be used with all four operations and with combinations of operations.

Multiplication Sequences

Here is a multiplication sequence called the *doubling sequence*.

(1, 2, 4, 8, 16, . . .)

Multiplication sequences are also called *geometric progressions*. The multiplication sequence below involves square numbers. It is the square of each of the counting numbers.

(1, 4, 9, 16, 25, 36, . . .)

Declining Sequences

Sequences with subtraction and division decline from higher to lower numbers.

(28, 25, 22, 19, 16, 13, . . .) The rule is: (n – 3).

(88, 44, 22, 11, . . .) The rule is: (n ÷ 2).

Harder Sequences

Some sequences use two operations like this one.

(5, 11, 23, 47, 95, . . .) The operations are multiply by 2 and add 1.

The rule can be written this way: (n x 2) + 1

This sequence also employs two operations.

(5, 9, 17, 33, 65, . . .) The operations are multiply by 2 and subtract 1.

The rule can be written this way: (n x 2) – 1

Fibonacci Sequences

The most famous sequence is called the *Fibonacci sequence*. The first two numbers are 1. Every number after that is the sum of the preceding two numbers. Sequences can be designed to follow this Fibonacci pattern.

(1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89)

Secret Sequences

Simple Sequences

A *sequence* is a set of numbers which follows a mathematical rule and a specific order.

Sample

(3, 6, 12, 24, 48, . . .) Each term after the first is multiplied by 2.

Directions: Study the Facts and Reminders page for this unit. Complete these sequences by filling in the blanks. Write the rule which the sequence follows. The first problem has been partially done for you.

1. (2, 4, 6, 8, 10, ____, ____, ____, ____) Rule: add 2 or $n + 2$

2. (1, 2, 3, ____, ____, 6, 7, ____, ____) Rule: _____

3. (9, 13, 17, ____, ____, 29, 33, ____, ____) Rule: _____

4. (6, 14, 22, 30, ____, ____, 54, 62, ____, ____) Rule: _____

5. (5, 10, 15, ____, ____, ____, 35, 40, ____) Rule: _____

6. (7, 10, 13, ____, ____, 22, 25, ____, ____) Rule: _____

7. (40, 38, 36, 34, ____, ____, ____, ____) Rule: _____

8. (132, 121, 110, 99, ____, ____, ____, ____) Rule: _____

9. (98, 93, 88, ____, ____, 73, 68, ____, ____) Rule: _____

10. (4, 10, 16, ____, ____, 34, ____, 46, ____) Rule: _____

11. (1, 2, 4, 8, ____, ____, ____, 128, ____) Rule: _____

12. (1, 3, 9, 27, ____, ____, ____, ____) Rule: _____

13. (3, 6, 12, ____, 48, ____, ____, ____) Rule: _____

14. (1, 4, 16, 64, ____, ____, ____, ____) Rule: _____

15. (5, 15, 45, ____, ____, ____, ____) Rule: _____

16. (1, 5, 25, ____, 625, ____, ____, ____) Rule: _____

17. Can you find the rule and complete this sequence?

(2048, ____, 512, ____, 128, ____, 32, ____) Rule: _____

Secret Sequences

Harder Sequences

Some sequences involve squared numbers or cubed numbers like the example below.

(1, 4, 9, 16, 25) Rule: counting numbers squared

Some sequences use two operations like the example below.

(1, 4, 13, 40, 121) The two operations are multiply by 3 and add 1. The rule can be written this way: (*n* x 3) + 1

Directions: Study the Facts and Reminders page for this unit. Complete these sequences by filling in the blanks. Write the rule which the sequence follows.

1. (2, 5, 11, 23, ____, ____, ____, ____) Rule: _____

2. (3, 10, 31, ____, ____, 850, ____, ____) Rule: _____

3. (1, 6, 26, 106, ____, ____, ____, ____) Rule: _____

4. (1, 2, 7, 32, 157, ____, ____, ____, ____) Rule: _____

5. (1, 4, 9, ____, ____, 36, ____, ____, ____) Rule: _____

6. (1, 8, 36, 148, ____, ____, ____, ____) Rule: _____

7. (4, 11, 32, 95, ____, ____, ____, ____) Rule: _____

8. (1, 5, 33, 229, ____, ____, ____, ____) Rule: _____

9. (5, 13, 29, 61, ____, ____, ____, ____) Rule: _____

10. (7, 15, 31, ____, 127, ____, ____, ____, ____) Rule: _____

11. (–10, –8, –5, –1, +4, +10, ____, ____, ____, ____) Rule: _____

12. (27, 26, 24, 21, 17, ____, ____, –1, –9, –18, ____) Rule: _____

13. (100, 90, 81, 73, ____, ____, ____, ____) Rule: _____

14. (2, 3, 5, 7, 11, 13, ____, ____, ____, ____) Rule: _____

15. (1, 8, 27, 64, ____, ____, ____, ____) Rule: _____

16. (–10, –5, +1, +8, +16, ____, ____, ____, ____) Rule: _____

17. (–30, –20, –11, –3, +4, +10, ____, ____, ____, ____) Rule: _____

18. (2, 5, 10, 17, 26, ____, ____, 65, ____, ____) Rule: _____

Secret Sequences

Fibonacci Sequence

The *Fibonacci sequence* is the most famous of all sequences in math. Every number after the first two 1s is computed by adding the two previous numbers.

(1, 1, 2, 3, 5, 8, 13, 21, 34, 55) Rule: add the previous two numbers

1 + 1 = **2** 1 + 2 = **3** 2 + 3 = **5** 3 + 5 = **8**

Directions: Answer the following questions.

1. Extend the Fibonacci sequence to 20 terms.

(1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___)

2. Compute the sum of the first ten numbers in the Fibonacci sequence. _____

3. Multiply the seventh term in the sequence (13) times 11. _____ You should get the same number. Double-check your work if your answers did not match.

Directions: Complete these Fibonacci sequences by filling in the blanks. Compute the sum of the first ten terms in each sequence. Multiply the seventh term in the sequence times 11.

4. (3, 3, 6, 9, 15, ____, ____, ____, ____, ____) Sum: _____

Product: 11 x ____ = _____

5. (7, 7, 14, 21, 35, ____, ____, ____, ____, ____) Sum: _____

Product: 11 x ____ = _____

6. (10, 10, 20, 30, 50, ____, ____, ____, ____, ____) Sum: _____

Product: 11 x ____ = _____

7. (5, 6, 11, 17, 28, ____, ____, ____, ____, ____) Sum: _____

Product: 11 x ____ = _____

8. (5, 5, 10, 15, 25, ____, ____, ____, ____, ____) Sum: _____

Product: 11 x ____ = _____

9. Use the Fibonacci sequence to answer this problem. If the first term of the sequence represented this year, what year would be represented by the fifth term? _____ What year would be represented by the eighth term? _____

Page 66

7. Prime numbers: 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199
9. 211
10. 1013
11. 997
12. 17 x 17 = 289
13. 19 x 19 = 361
14. 23 x 23 = 529

Page 68

1. (2, 4, 6, 8, 10, 12, 14, 16, 18)
   $n + 2$
2. (1, 2, 3, 4, 5, 6, 7, 8, 9)
   $n + 1$
3. (9, 13, 17, 21, 25, 29, 33, 37, 41)
   $n + 4$
4. (6, 14, 22, 30, 38, 46, 54, 62, 70, 78)
   $n + 8$
5. (5, 10, 15, 20, 25, 30, 35, 40, 45)
   $n + 5$
6. (7, 10, 13, 16, 19, 22, 25, 28, 31)
   $n + 3$
7. (40, 38, 36, 34, 32, 30, 28, 26)
   $n - 2$
8. (132, 121, 110, 99, 88, 77, 66, 55)
   $n - 11$
9. (98, 93, 88, 83, 78, 73, 68, 63, 58)
   $n - 5$
10. (4, 10, 16, 22, 28, 34, 40, 46, 52)
    $n + 6$
11. (1, 2, 4, 8, 16, 32, 64, 128, 256)
    $n \times 2$
12. (1, 3, 9, 27, 81, 243, 729, 2187)
    $n \times 3$
13. (3, 6, 12, 24, 48, 96, 192, 384)
    $n \times 2$
14. (1, 4, 16, 64, 256, 1024, 4096, 16384)
    $n \times 4$
15. (5, 15, 45, 135, 405, 1215, 3645)
    $n \times 3$
16. (1, 5, 25, 125, 625, 3125, 15625, 78125)
    $n \times 5$
17. (2048, 1024, 512, 256, 128, 64, 32, 16)
    $n \div 2$

Page 69

1. (2, 5, 11, 23, 47, 95, 191, 383)
   $(n \times 2) + 1$
2. (3, 10, 31, 94, 283, 850, 2551, 7654)
   $(n \times 3) + 1$
3. (1, 6, 26, 106, 426, 1706, 6826, 27306)
   $(n \times 4) + 2$
4. (1, 2, 7, 32, 157, 782, 3907, 19532, 97657)
   $(n \times 5) - 3$
5. (1, 4, 9, 16, 25, 36, 49, 64, 81)
   (counting numbers squared)
6. (1, 8, 36, 148, 596, 2388, 9556, 38228)
   $(n \times 4) + 4$
7. (4, 11, 32, 95, 284, 851, 2552, 7655)
   $(n \times 3) - 1$
8. (1, 5, 33, 229, 1601, 11205, 78433, 549029)
   $(n \times 7) - 2$
9. (5, 13, 29, 61, 125, 253, 509, 1021)
   $(n \times 2) + 3$
10. (7, 15, 31, 63, 127, 255, 511, 1023, 2047)
    $(n \times 2) + 1$
11. (–10, –8, –5, –1, +4, +10, +17, +25, +34, +44)
    (add +2, +3, +4, etc.)
12. (27, 26, 24, 21, 17, 12, 6, –1, –9, –18, –28)
    (subtract 1, 2, 3, etc.)
13. (100, 90, 81, 73, 66, 60, 55, 51)
    (subtract 10, 9, 8, etc.)
14. (2, 3, 5, 7, 11, 13, 17, 19, 23, 29) (prime numbers)
15. (1, 8, 27, 64, 125, 216, 343, 512)
    (counting numbers cubed)
16. (–10, –5, +1, +8, +16, +25, +35, +46, +58)
    (add +5, +6, +7, etc.)
17. (–30, –20, –11, –3, +4, +10, +15, +19, +22, +24)
    (add +10, +9, +8, etc.)
18. (2, 5, 10, 17, 26, 37, 50, 65, 82, 101)
    (counting numbers squared +1)

Page 70

1. (89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765)
2. 143
3. 143
4. (24, 39, 63, 102, 165)
   Sum: 429
   Product: 11 x 39 = 429
5. (56, 91, 147, 238, 385)
   Sum: 1001
   Product: 11 x 91 = 1001
6. (80, 130, 210, 340, 550)
   Sum: 1430
   Product: 11 x 130 = 1430
7. (45, 73, 118, 191, 309)
   Sum: 803
   Product: 11 x 73 = 803
8. (40, 65, 105, 170, 275)
   Sum: 715
   Product: 11 x 65 = 715
9. Answers will vary.

Page 72

1. 1, 2, 7, 14
2. 1, 2, 3, 4, 6, 12
3. 1, 2, 5, 10
4. 1, 2, 4, 8, 16
5. 1, 3, 5, 15
6. 1, 2, 4, 5, 10, 20
7. 1, 5, 25
8. 1, 2, 4, 8, 16, 32
9. 1, 3, 7, 21
10. 1, 2, 4, 5, 8, 10, 20, 40
11. 1, 2, 4, 11, 22, 44
12. 1, 2, 4, 7, 14, 28
13. Factors of 22: 1, 2, 11, 22
    Factors of 27: 1, 3, 9, 27
    Common Factors: 1
14. Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
    Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
    Common Factors: 1, 2, 3, 4, 6, 12
15. Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56
    Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42
    Common Factors: 1, 2, 7, 14
16. Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
    Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
    Common Factors: 1, 2, 4, 5, 10, 20

Page 73

1. Factors of 20: 1, 2, 4, 5, 10, 20
   Factors of 25: 1, 5, 25