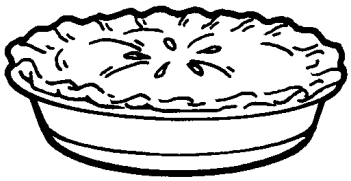
3 How to • • • • • Work Backwards Toward Answers

Facts to Know

To solve some problems, you may need to undo the key actions in the problem. This strategy is called working backwards. But before you start these problems, keep in mind that you need to ask yourself these four important questions:

- What does the problem ask you to find?
- What information is needed to solve the problem?
- How can you show the problem with numbers?
- What is your answer?

Sample A



The bakers at Slice o' Pie Restaurant ate 4 pies that were left over from work on Thursday night. 12 pies were sold at the restaurant that night. The manager took 2 home with her after work. How many pies were at the restaurant Thursday night?

First, find out the total number of pies sold or taken home.

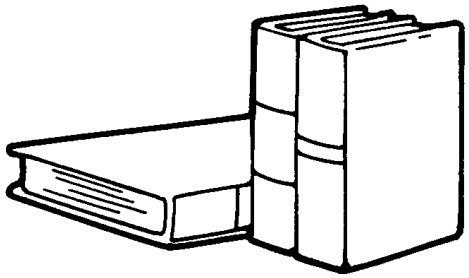
12 pies sold + 2 pies taken home = 14 pies

Next, add back the 4 pies that the bakers ate.

14 pies sold or taken home + 4 pies eaten by the bakers = 18 pies

Therefore, there were 18 pies at Slice o' Pie Restaurant Thursday night.

Sample B



Marta likes to borrow books from the library. She borrowed 18 books over a 3-month period of time (January-March). If Marta borrowed 8 books in January and 6 books in February, how many books did she borrow in March?

Subtract the number of books Marta borrowed in January and February from the total number of books to determine how many she borrowed in March.

18 books – 8 books – 6 books = 4 books

Marta borrowed 4 books in March.

3 Practice • • • • • • • • • • • • • • • Finding Solutions by Working Backwards

Directions: Using the information on page 13, solve the problems below and on pages 15 and 16. Some of the problems provide hints to help you solve them.

Grace has to be at work by 9:00 A.M. It takes her 15 minutes to get dressed, 20 minutes to eat, and 35 minutes to walk to work. What time should she get up?

1. What does the problem ask you to find?

a. what's the total time it takes her to get ready

b. how far away is work

c. what time does she have to be at work

d. what time should she get up

2. What information is needed to solve the problem?

e. what time does she get to work

f. what's the sum of (15 + 20 + 35) minutes

g. what's the difference between 9:00 A.M. and the sum of (15 + 20 + 35) minutes

h. what's the total sum of 9:00 A.M. and the sum of (15 + 20 + 35) minutes

3. How can you show the problem with numbers?

Hint: *Write a problem that shows working backward from 9:00 A.M. by subtracting the time Grace spent doing other activites to calculate what time she got up.*

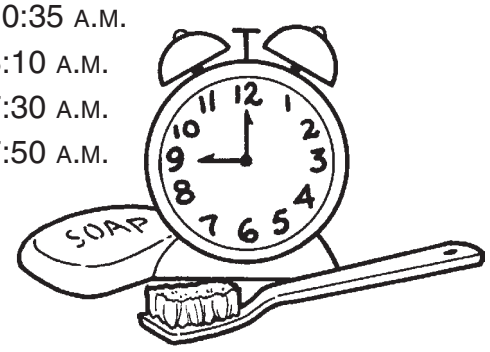
4. What is your answer?

i. 10:35 A.M.

j. 8:10 A.M.

k. 7:30 A.M.

l. 7:50 A.M.



Mr. Hall asked his students to open their math books to the facing pages whose page numbers add up to 85. To which pages should the children turn?

5. What does the problem ask you to find?

a. what is page 85

b. how to open the math book to facing pages

c. what are the facing pages whose page numbers add up to 85

d. how many students opened their books to page 85

6. What information is needed to solve the problem?

e. how many tries will it take to get 85

f. what is 1 x 85

g. what is 84 + 86

h. what is about half of 85

7. How can you show the problem with numbers?

Hint: *You know the total already. The numbers of two facing pages that total 85 must be around half. Try dividing by two, but remember that page numbers are whole numbers that increase from lowest to highest.*

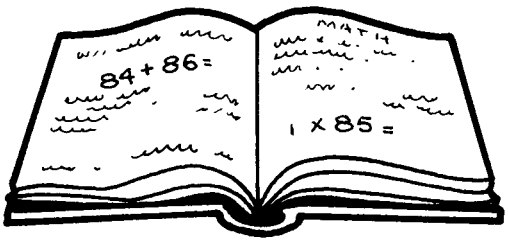
8. What is your answer?

i. 84 + 86

j. 40 + 45

k. 41 + 44

l. 42 + 43



3 Practice • • • • • • • • • • • • • • • Finding Solutions by Working Backwards

Kyle played the same game with Mr. Hall. "I'm thinking of two facing pages that total 127, Mr. Hall," he said. Kyle is thinking of which two facing pages?

9. What does the problem ask you to find?

a. what totals 127

b. what game are Kyle and Mr. Hall playing

c. what is half of 127

d. what are the page numbers of two facing pages that add up to 127

10. What information is needed to solve the problem?

e. how many tries will it take to get 127

f. what is 1 x 127

g. what is 126 + 128

h. what is about half of 127

11. How can you show the problem with numbers?

i. 127 – 1 =

j. (127 – 1) + (127 +1) =

k. 127 ÷ 2 =

l. 127 x 2 =

12. What is your answer?

m. 62 + 63

n. 63 + 64

o. 65 + 66

p. 64 + 65

Felicia passed around a basket of strawberries to the girls at her party. Before the party she ate 5 strawberries and gave a friend 3. Then 8 girls arrived at the party. The first girl took a strawberry, the second girl took 3 strawberries, the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. How many strawberries were in the basket at the beginning?

13. What does the problem ask you to find?

a. how many strawberries each girl ate

b. how many strawberries were in the basket at the beginning

c. how many strawberries were eaten before the party

d. how many girls were at the party

14. What information is needed to solve the problem?

Hint: *You need to find the total when eight girls were taking two more strawberries than the previous girl each time. Then you can add back in the 8 that were taken before the party.*

15. How can you show the problem with numbers?

e. 8 x 8 =

f. (1 + 3 + 5) x 8 =

g. 8 x 2 + 8 =

h. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + (3 + 5) =

16. What is your answer?

i. 128

j. 64

k. 72

l. 17

3 Practice • • • • • • • • • • • • • • • Finding Solutions by Working Backwards

In Wisconsin the Washington Island, ferryboat is full when it has 10 cars on board. It is also full when it has six trucks on board. The ferryboat never carries cars and trucks at the same time.

The ferryboat made five trips across the channel and was full on each trip. It ferried a total of 42 cars and trucks across.

How many cars did the ferryboat carry altogether in the five trips?

17. What does the problem ask you to find?

a. how many cars and trucks went across

b. how many cars and trucks can the ferryboat take at the same time

c. how many trucks went in five trips

d. how many cars went in five trips

18. What information is needed to solve the problem?

e. how many trucks went across in how many trips

f. how many trucks can go in five trips

g. what's the difference between 50 cars and 30 trucks

h. why doesn't the ferry carry cars and trucks at the same time

19. How can you show the problem with numbers?

i. 30 cars + 12 trucks

j. (5 x 6 cars) + (6 x 12 trucks)

k. 5 x 10 cars

l. 32 cars + 10 trucks

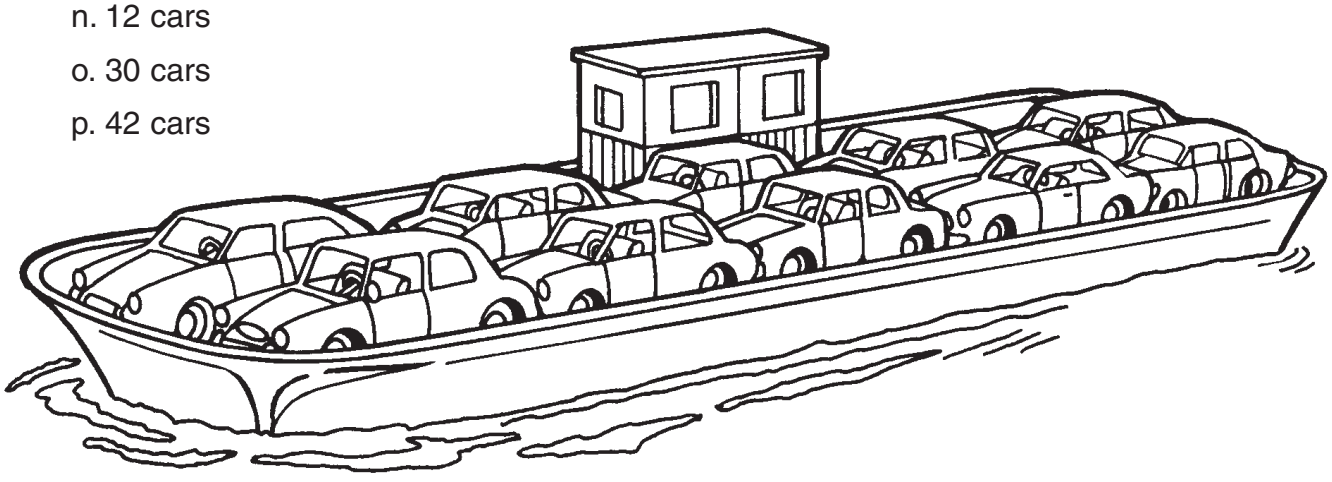
20. What is your answer?

m. 50 cars

n. 12 cars

o. 30 cars

p. 42 cars



Answer Key

Pages 7 and 8

1. Step #1: c
   Step #2: f
   Step #3: i
   Step #4: m
2. Step #1: a
   Step #2: e
   Step #3: j
   Step #4: o
3. Step #1: c
   Step #2: g
   Step #3: j
   Step #4: n

Pages 10–12

1. c
2. (given)
3. h
4. j

   1,753 (2nd year)
   − 152 (less 3rd year)
   1,601 (3rd year total)

   1,572
   1,753
   + 1,601
   4,926

5. d
6. g
7. 110–95
   130–95
   116–95
8. j
   110 – 95 = 15 cookies leftover
   130 – 95 = 35 fudge leftover
   116 – 95 = 21 peanut butter squares left over
9. d
10. g
11. (given)
12. j
    Each day the grasshopper goes 1/8 m until the day when the grasshopper is at 1.75 m in the morning. He gets out of the hole that day. 1.75/(1/8) = 14 and 14 + 1 more day = 15 days.
13. b
14. (given)
15. h
16. i
    7 (days) x $191,781 (per day) = $1,342,467
17. 24,000 kilometers/32 kilometers per gallon = 750 gallons in 1 year
    750 gallons x .05 = 37.5 gallons
    You could save 37.5 gallons in 1 year.
18. Convert the 5 pounds into ounces by multiplying 5 pounds by 16 ounces (1 pound). Five pounds is 80 ounces. There are 8 families. 80/8 is 10. Each family gets 10 ounces of cheese.
19. 1 billion seconds x 1 min/60 sec = 16,666,667 minutes
    16,666,667 minutes x 1 hour/60 min = 277,778 hours
    277,778 hours x 1 day/24 hours = 11,574 days
    11,574 days x 1 year/365 days = 32 years.
    A billion seconds is 32 years old.
20. 1 hour 10 minutes is 70 minutes.
    28 seedlings ÷ 40 min = .7 of a seedling per minute
    .7 of a seedling x 70 minutes = 49 seedlings

Pages 14–16

1. d
2. g
3. (given)
4. l
   9:00 A.M. – (15 minutes + 20 minutes + 35 minutes)
   9:00 A.M. – 70 minutes, or 1 hour 10 minutes = 7:50 A.M.
5. c
6. h
7. (given)
8. l
   85 ÷ 2 = 42.5, but page numbers go in order; pages 42 and 43
9. d
10. h
11. k
12. n
13. b
14. (given)
15. h
16. k
    1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64 strawberries
    64 + 8 (taken out earlier) = 72 strawberries at the start
17. d
18. e
19. i
20. o
    3 trips with 10 cars + 2 trips with 6 trucks
    30 cars + 12 trucks = 42 vehicles

Pages 19 and 20

1. 540 minutes
2. 1,000 – 1,200 people
3. front-end estimation
4. compatible numbers
5. $9,000 – $11,000
6. a. greater
   b. Henry 240; April 210; Felicia 260
   c. April and Felicia
   d. 115 and 120
7. 730
8. $1,000 – $1,200
9. front-end estimation
10. 700 (by rounding down)
11. 412 + 629 ~ 1000
    325 + 685 ~ 1000
    879 ~ 900
    estimate: 2,900
12. 1,400 – 1,600
13. Colleen 8, Andrea 5
14. 13 paved, 8 unpaved

Pages 22–24

1. d
2. h
3. i
4. m
5. c
6. g
7. (1,000 x 3/4) – 200 = 550
8. j
   3/4 of 1000 is 750 cars in on Tuesday. 750 cars – 200 compact cars = 550 standard-size cars
9. d
10. f
11. (given)
12. j
    t (time) = 1.0 miles (distance) ÷ 1/5 mile per min (rate) = 5 min
13. b
14. h
15. i
16. m
17. b
18. e
19. i
20. o
    If it costs 77 cents to produce 35 cars, then each car costs: 77/35 = 2.2 cents. It costs 2.2 cents to make each car. So 385 cars x 2.2 cents for each car = 847 cents, or $8.47 to produce 385 cars.

Page 28

1. 2 1/2 cookies each
2. $55.75
3. $14,756.75
4. $148.00
5. 16
6. 50, 100
7. 68
8. 225
9. $50
10. $32.50

Pages 30–32

1. First column 4 = 2 + 2
   2 x 2 = 4
   Second column 5 = 2 + 3
   2 x 3 = 6

4	5	9	**13**	**10**	**15**	16	**3**	7
2	2	4	**7**	5	7	**7**	1	**3**
2	3	5	6	**5**	8	**9**	2	4
4	6	20	42	25	**56**	63	2	12

2. 1,2 2,3 3,4 4,5 5,6 6,7 7,8 8,9
   1,3 2,4 3,5 4,6 5,7 6,8 7,9
   1,4 2,5 3,6 4,7 5,8 6,9
   1,5 2,6 3,7 4,8 5,9
   1,6 2,7 3,8 4,9
   1,7 2,8 3,9
   1,8 2,9
   1,9
   36 = 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

   There are 36 children.

3. a. 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 = $156.00

   b. $8190.00

4. .01 +.02 +.04 +.08 +.16 +.32 +.64 = $1.27 > $1.25

   Rob should ask for a daily allowance.

5. Every 7 days it's another Tuesday. So start by dividing 100 by 7 to get 14 with a remainder of 2. So in 100 days, 14 Tuesdays will go by + 2 extra days, making it Thursday.

6.

Renters	Kilowatt Hours
1	2
2	5
3	7
4	10
5	12
6	15
7	17
8	20
9	22
10	25

7. 320
8. 50 minutes
9. August 9th
10. Each member in the series is the sum of the two numbers before it.

    The next numbers are 377, 610, 987

11. East! It always points east at a quarter past.
12.

15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	0